Solution: HW #2

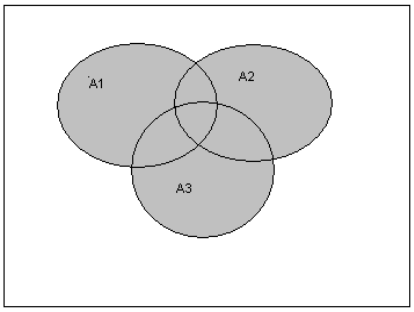
Title: Foundations of Econometrics Fall/2015
Course: Econ 367 Instructor: Dr. I-Ming Chiu

Q1. 8 (Exercise 2.1, pp. 56)
Answer:
a) A sequence of actions is taken by the student till the outcome is either 3, 4 or 5. Before 3, 4, or 5 are selected, there are four possible outcomes: "1", "2", "12" and "21". In total there 3*4 + 3 (3, 4, or 5 is selected in the first round) = 15.

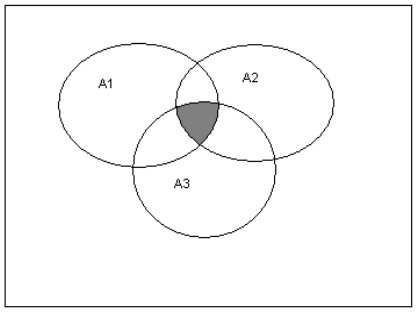
S = {3, 4, 5, 13, 14, 15, 23, 24, 25, 123, 124, 125, 213, 214,215}

b) A = {3, 4, 5}
c) B = {5, 15, 125, 215}
d) C = {3, 4, 5, 23, 24, 25}

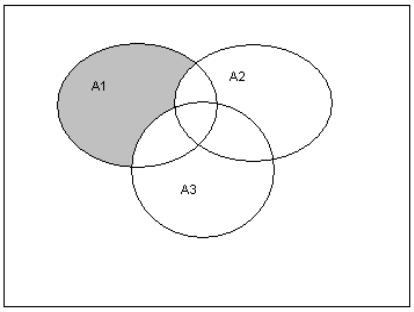
Q2. 10 (Exercise 2.1, pp. 56)
Answer:
a) $A_1 \cup A_2 \cup A_3$
b) $A_1 \cap A_2 \cap A_3$
c) $A_1 \cap A_2^C \cap A_3^C$
d) $(A_1 \cap A_2^C \cap A_3^C) \cup (A_1^C \cap A_2 \cap A_3^C) \cup (A_1^C \cap A_2^C \cap A_3)$
e) $A_1 \cup (A_2 \cap A_3)$



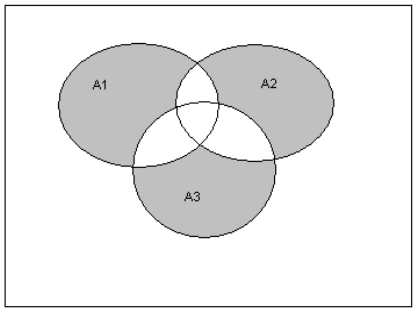
(a)



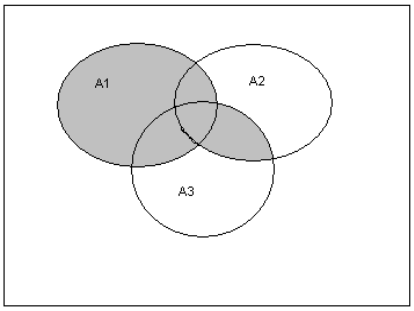
(b)



(c)



(d)



(e)

Q3. 11 (Exercise 2.1, pp. 56)
Answer:
I will draw both in our meeting.

Q4. 14 (Exercise 2.2, pp. 63)
Answer: Given condition: $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.25$
a) $P(A \cup B) = P(A) + P(B) – P(A \cap B) = 0.65$
b) $P(A^C \cap B^C) = P((A \cup B)^C) = 1 - P(A \cup B) = 0.35$
c) $P(A \cap B^C) = P(A) – P(A \cap B) = 0.25$

Q5. 22 (Exercise 2.2, pp. 64)
Answer:
a) Each cell in the table is a simple event; e.g., {Day and Unsafe conditions}
b) 23%
c) 1 - 10% - 35% = 55%

Q6. 24 (Exercise 2.2, pp. 65)
Answer:
A: stop at the 1st signal, B: stop at the 2nd signal
$P(A) = 0.4$, $P(B) = 0.5$, $P(A \cup B) = 0.6$

a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.3$
b) $P(A \cap B^C) = P(A) - P(A \cap B) = 0.1$
c) $P(A \cap B^C) \cup P(A^C \cap B) = 0.3$ Alternatively, it is equivalent to $P(A \cup B) - P(A \cap B)$

Q7. 40 (Exercise 2.3, pp. 73)
Answer:
Denote 40-, 60- and 75-W bulbs by A, B and C. There are 4As, 5 Bs, and 6 Cs.

a) $P(2\text{ Cs}) = \dfrac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}}$

b) $P(3\text{ As} \cup 3\text{ Bs} \cup 3\text{Cs}) = P(3\text{ As}) + P(3\text{ Bs}) + P(3\text{Cs}) = \dfrac{\binom{4}{3}}{\binom{15}{3}} + \dfrac{\binom{5}{3}}{\binom{15}{3}} + \dfrac{\binom{6}{3}}{\binom{15}{3}}$

c) $P(1\text{ A} \cap 1\text{ B} \cap 1\text{ C}) = \dfrac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}}$

d) To examine at least six items in order to obtain a 75W bulb == the first five bulbs can not be 75W bulbs

$\Rightarrow \dfrac{\binom{9}{5}}{\binom{15}{5}}$

Q8. 45 (Exercise 2.4, pp. 82)
Answer:
a) $P(A) = 0.447$, $P(C) = 0.50$, $P(A \cap C) = 0.2$
b) $P(A \mid C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{0.2}{0.5} = 40\%$, the percentage of type A in ethnic group 3.
$P(C \mid A) = \dfrac{P(A \cap C)}{P(A)} = \dfrac{0.2}{0.447} = 44.74\%$, the percentage of ethic group 3 among the type A
c) $0.082 + 0.106 + 0.004 = 0.192$
$P(B^C) = 1 - 0.091 = 0.909$
$0.192/0.909 = 21.12\%$

Q9. 75 (Exercise 2.5, pp. 89)

Answer:

System works = $P(1 \cup 2 \cup (3 \cap 4)) = P(1) + P(2) + P(3 \cap 4) - P(1 \cap 2) - P(1 \cap (3 \cap 4)) - P(2 \cap (3 \cap 4)) + P(1 \cap 2 \cap (3 \cap 4)) = 0.9 + 0.9 + 0.81 - 0.81 - 0.729 - 0.729 + 0.6561 = 0.9981$

Q10. (a) In the Bayes' theorem exercise in Handout #3, please use a Venn diagram to show the concept of true positive, false positive, and positive predictive value (PPV).
(b) Show what the negative predictive value (NPV) is using the conditional probability notation.
(c) Calculate the NPV by using the given information (i.e., prevalence rate, sensitivity, and specificity).

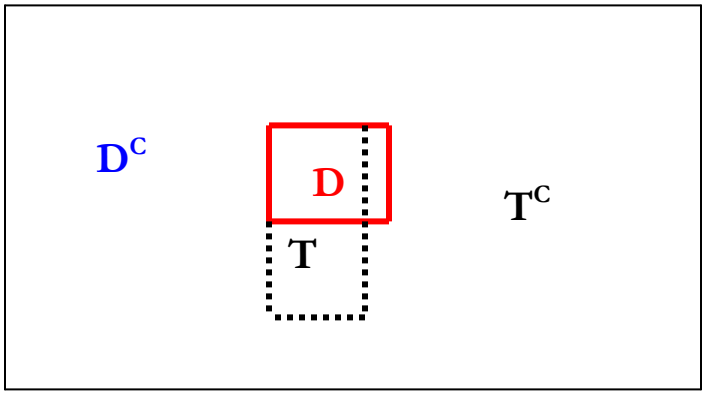
Answer:

a) Let's use D to denote disease and T for positive outcome. $P(D) = 0.005$

False positive = $P(T|D^C) = 0.02 \Leftrightarrow$ Specificity = $P(T^C|D^C) = 0.98$

False negative = $P(T^C|D) = 0.01 \Leftrightarrow$ Sensitivity = $P(T|D) = 0.99$

PPV = $P(D|T)$



b) NPV = $P(D^C|T^C)$

c) $\text{NPV} = \dfrac{P(T^C|D^C)*P(D^C)}{P(T^C)} = \dfrac{P(T^C|D^C)*P(D^C)}{P(T^C|D^C)*P(D^C)+P(T^C|D)*P(D)}$

$= \dfrac{0.98*0.995}{0.98*0.995+0.01*0.005} \cong 99.99\%$