Solution: HW #3

Title: Foundations of Econometrics Fall/2015
Course: Econ 367 Instructor: Dr. I-Ming Chiu

Note: I use either V or Var to denote variance

Q1. 4 (Exercise 3.1, pp. 100)
Answer:
a) X = 0, 1, 2, 3, 4, 5. If zip code = 08102, 12345, 00001, then X = 3, 5, 1, respectively. I am not sure whether this zip code, "00000", exists. If not, then X = 1, 2, 3, 4, 5.

Q2. 10 (Exercise 3.1, pp. 100)
Answer: $T = \{x: x \leq 10, x \in Z_+\}$; $X = \{x: -6 \leq x \leq 6, x \in Z\}$; $T = \{x: 0 \leq x \leq 6, x \in Z_+\}$; $Z = \{0, 1, 2\}$

Q3. 12 (Exercise 3.2, pp. 109)
Answer:
a) In order for the flight to accommodate all the ticketed passengers who show up, no more than 50 can show up. We need $Y \leq 50$.
$P(Y \leq 50) = .05 + .10 + .12 + .14 + .25 + .17 = .83$
b) Using the information in a. above, $P(Y > 50) = 1 - P(Y \leq 50) = 1 - .83 = .17$
c) For you to get on the flight, at most 49 of the ticketed passengers must show up. $P(Y \leq 49) = .05 + .10 + .12 + .14 + .25 = .66$. For the 3rd person on the standby list, at most 47 of the ticketed passengers must show up. $P(Y \leq 47) = .05 + .10 + .12 = .27$

Q4. 22. ((Exercise 3.2, pp. 111)
Answer:
a) $P(X = 2) = .39 - .19 = .20$
b) $P(X > 3) = 1 - .67 = .33$
c) $P(2 \leq X \leq 5) = .97 - .19 = .78$
d) $P(2 < X < 5) = .92 - .39 = .53$

Q5. 31 (Exercise 3.3, pp. 119)
Answer:
a) $E(X) = \sum_{1}^{3} x * f(x) = 16.38$; $E(X^2) = \sum_{1}^{3} x^2 * f(x) = 272.30$; $Var(X) = E(X^2) - (E(X))^2 = 4$

b) $\sum_{1}^{3} h(x) * f(x) = \sum_{1}^{3} (25 * x - 8.5) * f(x)$ or $E(25*X - 8.5) = 25*E(X) - 8.5 = 401$

c) $Var(h(x)) = E(h(x)^2) - (E(h(x)))^2$ or $Var(25*X - 8.5) = 625*Var(X) = 2500$

$E(h(x)^2) = \sum_{1}^{3} h(x)^2 * f(x) = \sum_{1}^{3} (25 * x - 8.5)^2 * f(x)$ or $E(X - 0.01X^2) = 13.66$

d) $E(h(X)) = E(X – 0.1*) = E(X) – 0.18E(X^2)$

Q6. 37 (Exercise 3.3, pp. 120)

Answer: $E(h(X)) = E(\frac{1}{X}) = \sum_{1}^{6} \frac{1}{x_i} * \frac{1}{6} = 0.4083 > \frac{1}{3.5}$

I would gamble on it because its expected value is larger. However, I could be conservative (depends on utility function) and decides to take the certain payout because the risk preference may change.

Q7. 72 (Exercise 3.5, pp. 137)

Answer:

a) $V(X) = np(1-p)$; to have $V(X) = 0$, then $p = 0$ or 1

b) $V = np – np^2$, the necessary condition for finding optimal value is $\frac{dV}{dp} = n – 2np = 0$, $p = 0.5$. $\frac{d^2V}{dp^2} = -2n < 0$, so it's a maximal point.

Q8. 75 (Exercise 3.5, pp. 137)

Answer:

a) $P(B) = 0.2$ so $P(B^C) = 0.8$. We have a binomial random variable B (use debit card) with 100 (n) trials.

$E(B) = np = 20$, $Var(B) = np(1 – p) = 16$

b) Combine A & B and denote it X, $P(X) = 0.7$

$E(X) = 70$, $Var(X) = 21$

Q9. 95 (Exercise 3.7, pp. 151)

Answer:

a) $P(X \le 10) = 0.0108 \Rightarrow$ *display poisson(20, 10)*

b) $P(X > 20) = 0.4409 \Rightarrow$ *display 1 – poisson(20, 20)*

c) $P(10 \le X \le 20) = 0.5541 \Rightarrow$ *display poisson(20, 20) – poisson(20, 9)*

$P(10 < X < 20) = 0.4594 \Rightarrow$ *display poisson(20, 19) - poisson(20, 10)*

d) $sd = \sqrt{20}$, $P(20 - 2*\sqrt{20} < X < 20 + 2*\sqrt{20}) = P(11.06 < X < 28.94) = 0.9443$ (11~28)

Q10. 98 (Exercise 3.7, pp. 151)

Answer:

a) X is Binomial

$E(X) = 10$, $Var(X) = nP(1 – P) = 9.99$

b) We can use Poisson to approximate the Binomial rv.

$E(X) = Var(X) = 10$

$P(X > 10) = 1 – P(X \le 10) = 41.70\% \Rightarrow$ *display 1 – poisson(10, 10)*

$P(X = 0) = 4.54*10^{-5} \Rightarrow$ *display poissonp(10, 0)*