Solution: HW #4

Title: Foundations of Econometrics — Fall/2015
Course: Econ 367 — Instructor: Dr. I-Ming Chiu

Notice: Q1~8 (80%), Q9 (20%)

Q1. 1 (Exercise 4.1, pp. 169)
Answer:

f(x) = 0.5*x, (domain or support: $0 \le X \le 2$) $\Rightarrow$ CDF(X) = $\int_0^x 0.5 * y(dy)$ = $0.25*x^2$

$P(X \le 1) = 0.25$
$P(0.5 \le X \le 1.5) = 0.25*1.5^2 - 0.25*0.5^2 = 0.5$
$P(1.5 < X) = 1 - P(X \le 1.5) = 1 - 0.25*1.5^2 = 0.4375$

Q2. Suppose the CDF for random variable X is shown as the following

$$CDF(X) = \frac{1}{8}X + \frac{3}{16}X^2, \text{ where } 0 \le X \le 2$$

a) Find $P(0.5 \le X \le 1.5)$.
b) Find the median of this random variable.
Answer:

a) $P(0.5 \le X \le 1.5) = \Phi(1.5) - \Phi(0.5) = (\frac{1}{8}*1.5 + \frac{3}{16}*1.5^2) - (\frac{1}{8}*0.5 + \frac{3}{16}*0.5^2) = 0.5$

b) Equate $\Phi(X) = 0.5 \Rightarrow \frac{1}{8}X + \frac{3}{16}X^2 = 0.5$, solve for X and X = 4/3 (the other answer, -2, is not in the domain).

Q3. 11 (Exercise 4.1, pp. 170) --- there is no need to use Stata for this one
Answer:
The CDF of this question reconfirms what we found in Q1.
a) $P(X \le 1) = \Phi(1) = 0.25$
b) $P(0.5 \le X \le 1) = \Phi(1) - \Phi(0.5) = 0.1875$
c) $P(X > 0.5) = 1 – P(X \le 0.5) = 1 - \Phi(0.5) \cong 0.9375$
d) $0.25*x^2 = 0.5 \Rightarrow x = \sqrt{2}$ (Notice: $-\sqrt{2} \notin$ domain)
e) 0.5*x

Q4. 40 ((Exercise 4.3, pp. 191) … use Stata, provide the code and outcome
Answer:
a) *display invnormal(0.9838)*
#2.14

b) normal(c) – normal(0) = 0.291
so normal(c) = 0.791 (*display normal(0) + 0.291*)
display invnornal(0.791)
#0.81
c) $P(c \le Z) = 0.121$, so $P(Z < c) = 0.879$
display invnorm(0.879)
#1.17
d) *display invnorm(0.332)*
#-0.97
So c = 0.97
e)
display invnorm(0.008)
#-2.41
So c = 2.41

Q5. 56 (Exercise 4.3, pp. 193) … use Stata, provide the code and outcome
Answer:

$P(X \le c - 1) = P(\frac{X-12}{3.5} \le \frac{(c-1)-12}{3.5}) = 0.99$

display invnorm(0.99)
#2.33
c – 13= 3.5*2.33, so c = 21.16

Q6. 6 (Exercise 6.1, pp. 295): Class presentation on this one

Q7. 11 (Exercise 6.2, pp. 304)
Answer:

In the handout, it shows that if $X \sim (\mu, \sigma^2) \Rightarrow \overline{X} \sim (\mu, \frac{\sigma^2}{n})$

a) $sd_X = \frac{\sigma}{\sqrt{n}} = \frac{0.04}{\sqrt{16}} = 0.01$

b) $sd_X = \frac{\sigma}{\sqrt{n}} = \frac{0.04}{\sqrt{64}} = 0.005$

c) (b), because the spread is much smaller (0.005 < 0.01) due to a larger sample (64 > 16).

Q8. Use examples, one for each, to explain the Law of Large Numbers and Central Limit Theorem.
Answer:
Law of large Numbers: the expected value of throwing a fair die is 3.5.
CLM: The sampling distribution of sample mean is normally distributed even the samples are drawn from non-normal random variable such as exponential distribution.

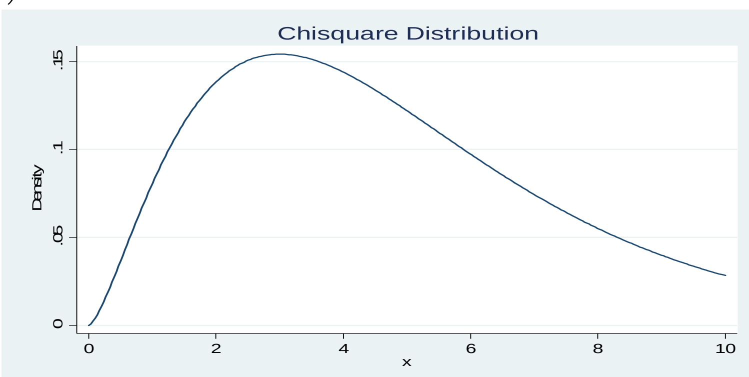
Q9. Suppose X_is are independent normal random variables with mean 0 and variance one. Please plot the following random variables in Stata.

a) $\sum_{i=1}^{5} X_i^{2}$; (b) $\frac{X_0}{\sqrt{\sum_{i=1}^{10} X_i^{2}/10}}$; (c) $(\sum_{i=1}^{5} X_i^{2}/5)/(\sum_{i=1}^{3} X_i^{2}/3)$

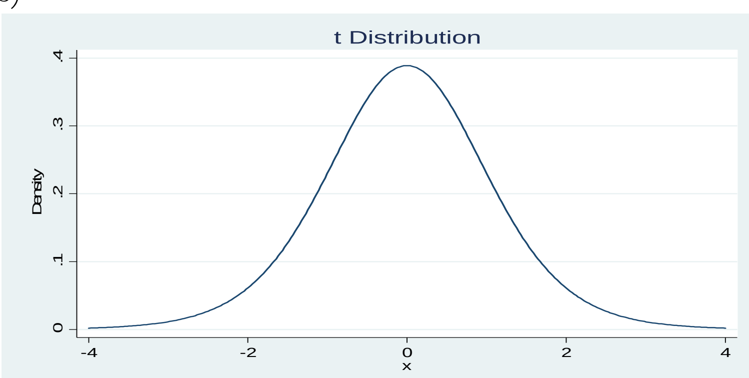
Answer:

As you can see, the first rv is Chi-square with df five. The second rv is t with df ten. The last rv is F with dfs five and three, respectively.

a)



b)



c)