Supplement to Handout #3

Title: FAE | Spring/2015
Course: Econ 368 | Instructor: Dr. I-Ming Chiu

A Brief Review to Probability Theory

(a) Interpretation of Probability

What's probability?
Answer: A numerical description of uncertainty.

Frequentist vs. Bayesian view

(b) Axioms of Probability

Three components in a probability space according to Russian mathematician A. N. Kolmogorov:

(a) A Sample Space
(b) Events
(c) A Probability Measure

The collection of events must satisfy the following property, called σ-field:

(a) The sample space is an event
(b) If E is an event, then E^C is an event
(c) The union of any countable collection of events is an event

e.g. Toss a coin twice

S (sample space) = {HH, HT, TH, TT}

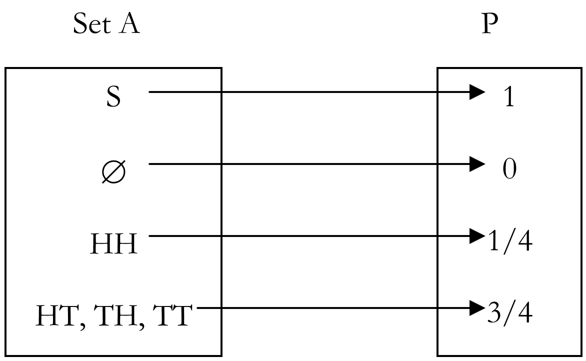
A = {S, ∅, HH, (HT, TH, TT})}
B = {S, ∅, (HT, TH)}

Is set A a σ-field? Is set B a σ-field?

The probability measure P must satisfy the following properties:

(a) If E is an event, then $0 \le P(E) \le 1$
(b) $P(S) = 1$
(c) If $E_1, E_2, \ldots$ is a countable collection of pair-wise disjoint events, then
$P(\cup E_i) = \sum P(E_i)$

e.g.



Theorem

(1) If E is an event $\Rightarrow P(E^C) = 1 - P(E)$

(2) If A and B are events and $A \subset B \Rightarrow P(A) \leq P(B)$

(3) If A and B are events and $A \subset B \Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(c) Finite Sample Spaces

$P(A) = \dfrac{\#A}{\#S}$ (the hash "#" symbol means "to count"; #A: count elements in set A)

By now, you should know why we need to learn some counting rules in Handout #3.
e.g. Throw a fair dice, what is the probability that the number on the top face is no greater than four?

e.g. A deck of 40 cards, labeled 1, 2, 3,…, 40, is shuffled and one hand of four cards is dealt to Arlen.
(a) What is the probability that Arlen's hand contains four even numbers?
(b) What is the probability that this hand is a straight?

e.g. The famous birthday problem: In a class of 30 students, what is the probability that at least two students share a common birthday?

(d) Conditional Probability, Law of Total Probability, and Bayes Theorem

(d-1) Conditional Probability

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} \Leftrightarrow P(A \cap B) = P(A|B)*P(B) = P(B|A)*P(A)$

What does the above conditional probability mean exactly?

(d-2) Law of Total Probability

If $A_1, A_2, \ldots, A_n$ are collectively exhaustive and mutually exclusive, then

$$P(B) = \sum_{i=1}^{n} P(B \mid A_i) * P(A_i)$$

Collectively exhaustive $\Rightarrow \cup A_i = S$
Mutually exclusive $\Rightarrow A_i \cap A_j = \phi$

e.g. Throw a fair dice, what is the probability to observe a six given that the outcome is an even number?

e.g. Three bags:
Bag1: 1W, 3R, Bag 2: 2W, 2R, Bag 3: 4W
(a) Pick a bag at random and then select a ball at random, what is the probability to choose a red ball from bag 2?
(b) Pick a bag at random then select two balls without replacement, what is the probability that both are red?
Answer:
(a) Denote bag 2 (B2) as event A and red ball (R) as event B, then the question asks to find what's the probability of $P(A \cap B)$?

We can use formula: $P(A \cap B) = P(A|B)*P(B) = P(B|A)*P(A)$, but which one?

Substitute B2 and R for A and B respectively;
$P(B2 \cap R) = P(B2|R)*P(R) = P(R|B2)*P(B2)$, so which one do you choose?

(b) Let B denote the event that there are 2 red balls (2R) and A_i denote the bag i (Bagi) is chosen.

$$P(2R) = \sum_{i=1}^{3} P(2R \mid Bagi) * P(Bagi)$$

Let's try one more example
e.g. There are four coins in a drawer. Two of them are fair, one has probability of 1/3 coming up heads, and the remaining one has the probability 3/4 coming up heads. Pick a coin at random and flip, what's the probability it comes up heads?

Independence
Def: A and B are independent $\Leftrightarrow P(A \cap B) = P(A)*P(B)$

e.g. A fair coin is tossed three times. A: a head occurs on each of the first two tosses. B: a tail occurs on the third toss. C: exactly two tails occur in the three tosses.
(a) Are event A and B independent?
(b) Are event B and C independent?

(d-3) Bayes Theorem

If $A_1, A_2, \ldots, A_n$ are collectively exhaustive and mutually exclusive, then

$$P(A_i | B) = \frac{P(B | A_i) * P(A_i)}{\sum_{j=1}^{n} P(B | A_j) * P(A_j)}$$

Let consider a simple case when j =2

$$P(A | B) = \frac{P(B | A) * P(A)}{P(B | A) * P(A) + P(B | A^C) * P(A^C)}$$

Does the denominator of the above formula look familiar to you?

e.g. Disease Testing

Let D = the subject has disease, T = the test reads positive, and P(D) = 0.5%

Q: We have a test which is 99% sensitive and 98% specific. If we apply the test to an individual and get a positive reading, what's the probability that he has the disease?

Definition:

Sensitivity = $P(T | D)$
Specificity = $P(T^C | D^C)$

What does this question ask? Can we show it in symbols & in a Venn diagram?
Term: PVP (Positive value predictive)