Handout #5

Title: FAE Spring/2015
Course: Econ 368/01 Instructor: Dr. I-Ming Chiu

Introduction to the Design of Experiments (DOX) (Reading: FCDAE, Chapter 1~3)

In handout one, we learned that data can be either observational or experimental. Using observational data, we can detect associations between variables; however, it is difficult to build causal relationships. One of the advantages of using the experimental data is it's collected in a controlled environment, and therefore, a causal relationship can be established between causes and effects. Design of experiments helps us establish causality between response (Y) and treatments (Xs) given that the confounding effects can be reduced to the minimum. While we will use the examples and the corresponding data from the FCDAE and BR books mostly, I will provide a simulated data set to examine the effectiveness of three learning environments in principles of economics classes; namely, on-site, on-line, and hybrid.

Components of an experiment:

a) Treatments.
b) Experiment units.
c) Response.
d) Assignment method.

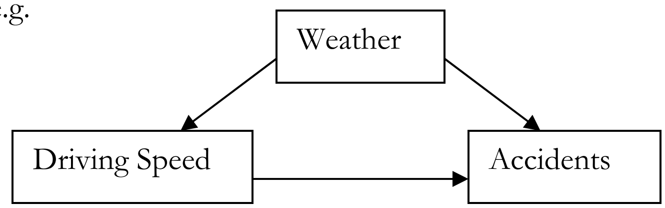
e.g. Does short-term incarceration of spouse abusers deter future assaults?

Treatments: warning, counseling but not booked on charges, and arresting.
Experiment units: individuals who assault their spouses.
Response: the length of time until recurrence of assault.
Assignment method: randomization.

Notice: DOX is not about the statistical analysis; it is about how we collect data for further analysis.

Randomization: an experiment is *randomized* if the method for assigning treatments to units involves a known, well-understood probabilistic scheme. The probabilistic scheme is called a *randomization*.

Randomization reduces confounding. What's confounding?
e.g.



e.g. new drug treatment vs. bypass surgery for coronary artery disease.
e.g. productive of two variety of corn; one is planted in Wisconsin and the other is planted in Iowa.

Completely Randomized Designs (CRD)

Structure of a CRD
We have g treatments to compare and N units to use in our experiment. For a completely randomized design:

1. Select sample sizes $n_1, n_2, \ldots, n_g$ with $n_1 + n_2 + \ldots + n_g = N$.
2. Choose n_1 units at random to receive treatment 1, n_2 units at random from the $N - n_1$ remaining to receive treatment 2, and so on.

This randomization produces a CRD; all possible arrangements of the N units into g groups with sizes n_1 though n_g are equally likely. Note that complete randomization only addresses the assignment of treatments to units; selection of treatments, experimental units, and responses is also required.

Model: A model for the data is a specification of the statistical distribution for the data (sampling scheme).

Parameter: Statistical distribution depends on parameters.
e.g. Binomial (n, θ), Normal (μ, σ^2)

Experimental data:
a) Model for the mean
b) Model for the error

Model 1: Separate means model

$Y_{i,j} = \mu_i + \varepsilon_{i,j} = \mu + \alpha_i + \varepsilon_{i,j}$ (equivalently; $Y_{i,j} \sim N(\mu_i, \sigma^2)$)

Model 2: Single mean model

$Y_{i,j} = \mu + \varepsilon_{i,j}$ (equivalently; $Y_{i,j} \sim N(\mu, \sigma^2)$)

Single mean model is a "reduced" model; it's a special case of the group means model.

Estimates of parameters in both models

Notations:

$Y_{i\bullet} = \sum_{j=1}^{n_i} Y_{i,j}$ (group total)

$Y_{\bullet\bullet} = \sum_{i=1}^{g}\sum_{j=1}^{n_i} Y_{i,j} = \sum_{i=1}^{g} Y_{i\bullet}$ (grand total)

$\overline{Y}_{i\bullet} = Y_{i\bullet}/n_i = \sum_{j=1}^{n_i} Y_{i,j}\,/n_i$ (treatment mean)

$\overline{Y}_{\bullet\bullet} = Y_{\bullet\bullet}/N$ (grand mean)

$\hat{\mu}_i = \overline{Y}_{i\bullet}$

$\hat{\mu} = \overline{Y}_{\bullet\bullet}$

$\hat{\alpha}_i = \hat{\mu}_i - \hat{\mu} = \overline{Y}_{i\bullet} - \overline{Y}_{\bullet\bullet}$ (treatment effects)

$Y_{i,j} - \mu = \alpha_i + \varepsilon_{i,j}$

Analysis of Variance (One way ANOVA): ANOVA can be considered as an extension of two-sample t-test; I will review it in our end of lecture exercises. Please refer to the BR book, pp. 193~203, for a variety of two-sample t-tests.

$Y_{i,j} - \overline{Y}_{\bullet\bullet} = (\overline{Y}_{i\bullet} - \overline{Y}_{\bullet\bullet}) + (Y_{i,j} - \overline{Y}_{i\bullet})$

$$\sum_{i=1}^{g}\sum_{j=1}^{n_i}(Y_{i,j} - \overline{Y}_{\bullet\bullet})^2 = \sum_{i=1}^{g}\sum_{j=1}^{n_i}(\overline{Y}_{i\bullet} - \overline{Y}_{\bullet\bullet})^2 + \sum_{i=1}^{g}\sum_{j=1}^{n_i}(Y_{i,j} - \overline{Y}_{i\bullet})^2$$

$$SS_T = SS_{Trt} + SS_E$$

Where $SS_{Trt} = \sum_{i=1}^{g}\sum_{j=1}^{n_i}(\overline{Y}_{i\bullet} - \overline{Y}_{\bullet\bullet})^2 = \sum_{i=1}^{g} n_i(Y_{i\bullet} - \overline{Y}_{\bullet\bullet})^2 = \sum_{i=1}^{g} n_i\hat{\alpha}_i^{\,2}$

ANOVA Table

Source	DF	SS	MS	F
Treatments	g - 1	SS_{Trt}	SS_{Trt}/g-1	MS_{Trt}/ MS_E
Error	N - g	SS_E	SS_E/N-g	

H_0: there is no treatment effect

H_A: otherwise

Decision rule: Reject null if F is large.

Exercise 3.2 (FCDAE, pp. 60)

An experimenter randomly allocated 125 male turkeys to five treatment groups: control and treatments A, B, C, and D. There were 25 birds in each group, and the mean results were 2.16, 2.45, 2.91, 3.00, and 2.71, respectively. The sum of squares for experimental error was 153.4. Test the null hypothesis that the five group means are the same against the alternative that one or more of the treatments differs from the control.

One-Way ANOVA

$SS_T = SS_{Trt} + SS_E$

Based on the information given in the question, we can calculate both SS_{Trt} and SS_E and then conduct an F test. (Why do we use F test?)

Correction: $\hat{\alpha}_i = \overline{Y}_{i\bullet} - \overline{Y}_{\bullet\bullet}$; $\overline{Y}_{i\bullet}$ are given in the question and $\overline{Y}_{\bullet\bullet}$ = (2.16+2.45+2.91+3+2.71)/5 = 2.646

$\hat{\alpha}_i = \hat{\alpha}_i = \overline{Y}_{i\bullet} - \overline{Y}_{\bullet\bullet} \Rightarrow$ –0.486, –0.196, 0.264, 0.354, and 0.064

$SS_{Trt} = \sum_{i=1}^{5} n_i \hat{\alpha}_i^{\,2} = 25*(-0.486^2 + -0.196^2 + 0.264^2 + 0.354^2 + 0.064^2) = 11.843$

$SS_E = 153.4$

$SS_{Trt} = 11.843$

$$F_{Obs} = \frac{MS_{Trt}}{MS_E} = \frac{SS_{Trt}/4}{SS_E/120} = \frac{11.843/4}{153.4/120} \approx 2.316$$

Since $F_{Obs} < F_{0.95,4,120}$($\approx$ 2.4472) or the p-value = 0.061, therefore, we cannot reject the null hypothesis that group means are the same.

ANOVA Table

Source	DF	SS	MS	F
Treatments	4	11.843	2.96075	2.316
Error	120	153.4	1.27833	
Total	124	165.243		

Use R for ONE-WAY ANOVA

The purpose is to test whether the means of multiple samples (>2) are the same.

$Y_{i,j} = \mu + \alpha_i + \varepsilon_{i,j}$

Where i indicates levels (factors) and j indicates observations. Later you will find that the above equation is similar to linear regression model, the difference is the type of explanatory variable (quantitative or qualitative). If there are both types of explanatory variables, the model is termed ANCOVA in some fields. Economists don't use this term.

We want to test

H_0: $Y_{i,j} = \mu + \varepsilon_{i,j}$

H_1: $Y_{i,j} = \mu + \alpha_i + \varepsilon_{i,j}$

We'll do many R exercises in the class meeting. I won't prepare the R code file for this lecture and the goal is to see whether you're familiar with some basic R usages.

e.g. diet and blood coagulation

```
> library("faraway")
> data(coagulation)
> names(coagulation)
> attach(coagulation)
> plot(coag~diet)
> Model1 = lm(coag~diet,coagulation)
> summary(Model1)
Call:
lm(formula = coag ~ diet, data = coagulation)

Residuals:
  Min     1Q      Median   3Q      Max
 -5.00    -1.25   0.00     1.25    5.00

Coefficients:
              Estimate     Std. Error   t value   Pr(>|t|)
(Intercept)  6.100e+01    1.183e+00    51.554    < 2e-16 ***
dietB        5.000e+00    1.528e+00    3.273     0.003803 **
dietC        7.000e+00    1.528e+00    4.583     0.000181 ***
dietD        -3.333e-15   1.449e+00    0.000     1.000000
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.366 on 20 degrees of freedom
```

Multiple R-squared: 0.6706, Adjusted R-squared: 0.6212
F-statistic: 13.57 on 3 and 20 DF, p-value: 4.658e-05

> Model1null = lm(coag~1,coagulation)
> anova(Model1null,Model1)

Analysis of Variance Table

Model 1: coag ~ 1
Model 2: coag ~ diet

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	23	340				
2	20	112	3	228	13.571	4.658e-05 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Exercises:

(a) Please use the "BodyTemperature.txt" data file to analyze whether "Gender" is a contributing factor to the "Temperature" (review two-sample t-test)
(b) Please use "econlearning.csv" data file to examine whether learning environments contribute to different learning outcomes.

*For those who are interested in two-way ANOVA or n-way ANOVA, please read FCDAE or BR to learn how to deal with two (or n) treatments.